BTW MODEL IN $d = 1$

Although the BTW model was originally introduced in two dimensions [1], it can also be defined in other dimensions including $d = 1$. Consider a system of size L with a number of grains h_i at each site i. Grains are deposited at the left-hand end and may leave the system at the right-hand end. When a grain arrives at a site i, $h_i \to h_i + 1$, it either remains there or topples on the next site to the right, $h_i \to h_i - 1; h_{i+1} \to h_{i+1} + 1$, depending on whether the slope $z_i = h_i - h_{i+1}$ exceeds some threshold z_{th}. The value of the threshold is not terribly important, so let us consider $z_{\text{th}} = 1$ as in the animation. Let us also follow the convention that $h_{L+1} = 0$, so that we can always define a slope z_L. The system is only driven, $h_1 \to h_1 + 1$, all the while grains are not toppling within the system. In this way, there is a timescale separation between the driving and relaxtional dynamics of the system. This, together with the existence of thresholds that can 'store' potential energy, is characteristic of self-organised critical systems.

As can be seen in the animation, the sandpile builds up in time until the staircase configuration is reached, after which the system is forever trapped in that recurrent configuration. Admittedly, the BTW model in $d = 1$ is somewhat trivial and does not display the richness of usual self-organised critical systems (for example, all avalanches are of size L in the staircase configuration). However, it can nevertheless be described in the same framework as the more non-trivial models, e.g. the avalanche size distribution can be written in terms of a scaling function [2]. Also, the BTW model in $d = 1$ can be used as a starting point for creating more sophisticated models, such as the Oslo model [3].

References

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