3. *Density of fractal object*

(i) (a) The mass $M = 8^n$ while the length $\ell = 3^n$.

$$M(\ell) = \ell^D \Leftrightarrow D = \frac{\log M(\ell)}{\log \ell} = \frac{n \log 8}{n \log 3} = \frac{\log 8}{\log 3} \approx 1.89.$$

(b) The density is by definition the mass per volume (here area), that is,

$$\rho(\ell) = \frac{M(\ell)}{\ell^d} = \frac{\ell^D}{\ell^d} = \ell^{D-d} = \ell^{-0.11}.$$

As the exponent is negative, the density will go to zero in the limit of large length scales

$$\lim_{\ell\to\infty} \rho(\ell) = \lim_{\ell\to\infty} \ell^{-0.11} = 0.$$

(c)

$$\text{Mass of black squares} + \text{Mass of white squares} = \ell^2 \quad \Leftrightarrow$$

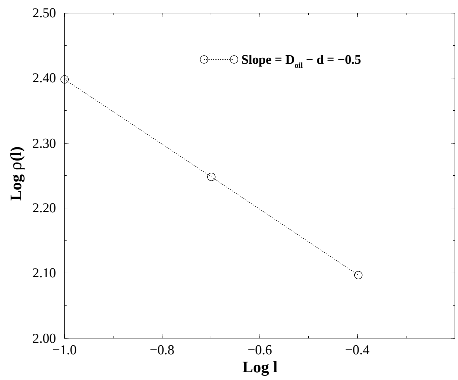
$$\ell^D + \text{Mass of white squares} = \ell^2 \Leftrightarrow \ell^{D-2} \quad + \quad \frac{\text{Mass of white squares}}{\ell^2} = 1.$$

Letting $\ell \to \infty$, the first term goes to zero, and we conclude

$$\text{Mass of white squares} \propto \ell^2 = \ell^d,$$

so the geometrical object of white squares will be compact.

(ii) (a) Plotting Log of $\ell = 0.1, 0.2, 0.4$ m versus Log of the density, we get a straight line with slope $D_{oil} - d = -0.5 \Leftrightarrow D_{oil} \approx 2.5$.



(ii) The length scale of the $d = 3$ oil field is $\ell_2 = 10000$ m. In order to calculate $M(\ell_2)$, we must determine the density $\rho(\ell_2)$ at length scale ℓ_2.

$$\rho(\ell_1) = A\ell_1^{D-d} \text{ and } \rho(\ell_2) = A\ell_2^{D-d} \Leftrightarrow \frac{\rho(\ell_2)}{\rho(\ell_1)} = \left(\frac{\ell_2}{\ell_1}\right)^{D-d} \Leftrightarrow$$

$$\rho(\ell_2) = \left(\frac{\ell_2}{\ell_1}\right)^{D-d} \cdot \rho(\ell_1) = \left(\frac{10000}{0.1}\right)^{-0.5} \cdot 250 \text{ kg m}^{-3} \approx 0.79 \text{ kg m}^{-3},$$

that is the amount of oil that can be recovered

$$M(\ell_2) = \ell_2^3 \cdot \rho(\ell_2) = 10000^3 \text{ m}^3 \cdot 0.79 \text{ kg m}^{-3} = 7.9 \cdot 10^{11} \text{ kg}.$$