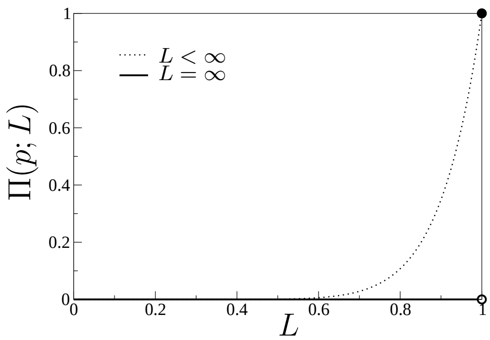
1.10 *Probability of having a percolating cluster on a lattice of size L.*

(i) Since for $p < p_c$ there is no percolating cluster, we have

$$\Pi_\infty(p, L = \infty) = \begin{cases} 0 & \text{for } p < p_c \\ 1 & \text{for } p = p_c. \end{cases}$$



(ii) (a) There is a percolating cluster only if all L sites are occupied. Therefore,

$$\Pi_\infty(p; L) = p^L,$$

see Figure above.

(b) We have

$$\Pi_\infty(\xi; L) = p^L = \exp\left(\ln p^L\right) = \exp\left(L \ln p\right) = \exp\left(-L/\xi\right)$$

using $\xi = -1/\ln p$.

(c) When $p \to p_c^-$, the correlation length $\xi \to (p_c - p)^{-1}$ so $1/\xi = (p_c - p)$. Therefore,

$$\Pi_\infty(\xi; L) \to \exp\left(-(p_c - p)L\right) = \mathcal{F}_{1\mathrm{d}}\left[(p_c - p)L\right] \quad \text{for } p \to p_c^-,$$

where we identify the scaling function $\mathcal{F}_{1\mathrm{d}}(x) = \exp(-x)$. Hence

$$\mathcal{F}_{1\mathrm{d}}(x) = \begin{cases} \text{constant} & \text{for } x \ll 1 \\ \text{decay rapidly} & \text{for } x \gg 1. \end{cases}$$

(iii) We assume that

$$\Pi_\infty(p; L) = \mathcal{G}\left(L/\xi\right) \quad \text{for } p \to p_c,$$

(a) Since $\xi \propto |p - p_c|^{-\nu}$ we have

$$\Pi_\infty(p; L) = \mathcal{G}\left(L/\xi\right) = \mathcal{G}\left(L|p - p_c|^\nu\right) = \mathcal{F}\left(L^{1/\nu}|p - p_c|\right) \text{ for } p \to p_c,$$

so that $\mathcal{F}(x) = \mathcal{G}(x^{1/\nu})$.

(b) In higher dimension, $\Pi_\infty(p; L)$ will approach a step function at $p = p_c$ when $L \to \infty$. Hence, the limiting function $d\Pi_\infty/dp = \delta(p_c - p)$ is a delta-function at $p = p_c$ when $L \to \infty$.