Statistical Mechanics 2006–2007

- **Lectures**
  Blackett Laboratory, Level 1, LT 2.
- **Lecturer**
  Kim Christensen.
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  E-mail: k.christensen@imperial.ac.uk
- **Office hours**
  Tuesdays and Wednesdays 12:00 – 13:00

- **Rapid feedback sessions**
  Blackett Laboratory, Level 1, LT 2. Thursdays 13:00 – 13:50.
- **Vera Pancaldi**
  E-mail: vera.pancaldi@imperial.ac.uk
- **Exercises**
  - Exercises in book Chaper.Exercise.
  - Additional exercises 1–7, Exam May 2001 – 2006.

  Hand in work to UG administration office, Level 3
  before Wednesdays 14:00.

- **Course URL**
  - www.cmth.ph.ic.ac.uk/people/k.christensen/statmech/
  - webct1.imperial.ic.uk (will become completed during the course)
- **Course prerequisites**
  The course will be self-contained
- **Assessment**
  May 2007. Written exam of 2h duration.

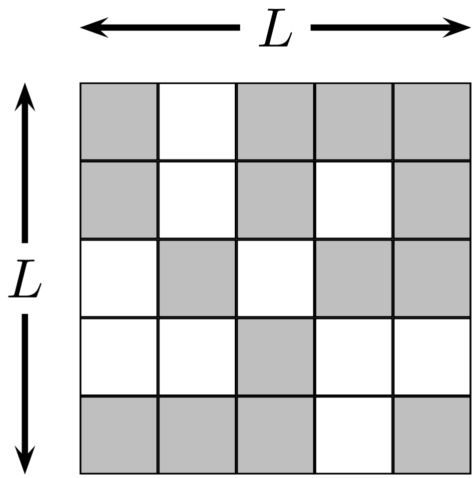
Statistical Mechanics

Connection between macroscopic and microscopic physics.

- **Introduction**: 1 lecture.
- **Percolation**: 9 lectures.
- **Ising model**: 9 lectures.
- **Self-organised criticality**: 7 lectures.

Aim: Study connections between macroscopic quantities and the underlying microscopic world in the simplest not exactly solved model displaying a phase transition.

Objective: Gain qualitative and quantitative understanding of the phenomenon of phase transition and associated concepts such as scale invariance, scaling theory, and universality.



Each site in a lattice is occupied randomly and independently with occupation probability p, where $0 \leq p \leq 1$.

A cluster is a group of nearest-neighbour occupied sites.

The critical occupation probability p_c is the occupation probability p at which an infinite cluster appears for the first time in an infinite lattice $L = \infty$.

Aim: Study connections between macroscopic phenomena and the underlying microscopic world for a ferromagnet.

How: Study the simplest possible model of a ferromagnet containing the essential physics: the Ising model.

Objective: Gain qualitative understanding of the physics governing the phenomena and reveal possible universal behaviour.

The simplest model of a ferromagnet consists of N spins $s_i = \pm 1 = \uparrow$ or $\downarrow$, $i = 1, \ldots, N$ with constant nearest-neighbour interaction $J > 0$. The energy of microstate $\{s_i\} = \{s_1, s_2, \ldots, s_N\}$

$$\begin{aligned} E_{\{s_i\}} &= \text{spin-spin interaction} \\ &= -J \sum_{\langle ij \rangle} s_i s_j . \end{aligned}$$

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The free energy $F = \langle E \rangle - TS$.

Aim: Study the connection between macroscopic phenomena and the underlying microscopic world for non-equilibrium systems.

How: Study simple non-equilibrium dynamical systems composed on many (locally) interacting particles.

Objective: To address why Nature is complex, not simple, as the laws of physics seem to imply.

Examples of complex behaviour in Nature: pattern formations, fractals, scale free distributions of event sizes for earthquakes, rain, solar flares, traffic congestions, price of cotton etc.

20^{th} century physics: If we understand the smallest objects, we understand everything. Like combining Lego blocks.

String theory $\rightarrow$ Particle physics

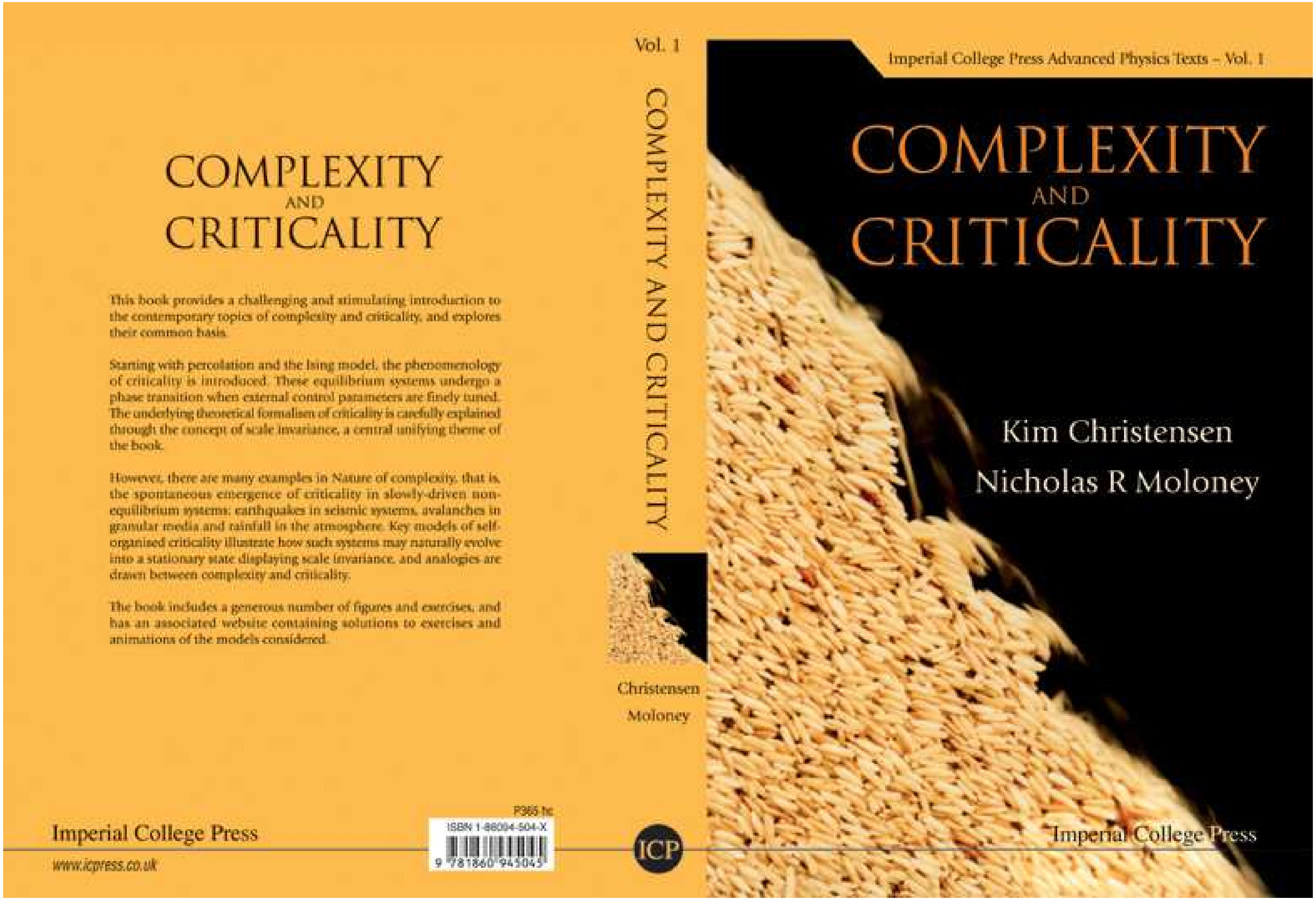
$\rightarrow$ Astrophysics, Geophysics

$\rightarrow$ Biology

$\rightarrow$ Humans

$\rightarrow$ History, Economics.

However, this is NOT true!



COMPLEXITY AND CRITICALITY

This book provides a challenging and stimulating introduction to the contemporary topics of complexity and criticality, and explores their common basis.

Starting with percolation and the Ising model, the phenomenology of criticality is introduced. These equilibrium systems undergo a phase transition when external control parameters are finely tuned. The underlying theoretical formalism of criticality is carefully explained through the concept of scale invariance, a central unifying theme of the book.

However, there are many examples in Nature of complexity, that is, the spontaneous emergence of criticality in slowly-driven non-equilibrium systems: earthquakes in seismic systems, avalanches in granular media and rainfall in the atmosphere. Key models of self-organised criticality illustrate how such systems may naturally evolve into a stationary state displaying scale invariance, and analogies are drawn between complexity and criticality.

The book includes a generous number of figures and exercises, and has an associated website containing solutions to exercises and animations of the models considered.

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